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Asymmetric jet correlations in $p\,p^{\uparrow}$ scattering

Daniël Boer*
Dept. of Physics and Astronomy, Vrije Universiteit Amsterdam,
De Boelelaan 1081, 1081 HV Amsterdam, The Netherlands

Werner Vogelsang†
Physics Department and RIKEN-BNL Research Center
Brookhaven National Laboratory, Upton, NY 11973, U.S.A.
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We propose that back-to-back correlations in azimuthal angle of jets produced in collisions of unpolarized with transversely polarized proton beams could be used to determine Sivers functions. The corresponding single-spin asymmetry is not power-suppressed, but is subject to Sudakov suppression. We present estimates of the asymmetry (without and with Sudakov effects) for RHIC at jet transverse momenta of ~ 10 GeV and show that it may reach a few per cent or more and could provide access to the gluon Sivers function.

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I. INTRODUCTION

An important goal of ongoing experiments with polarized protons at BNL's Relativistic Heavy-Ion Collider RHIC is to contribute to a better understanding of transverse-spin effects in QCD. Of particular interest are single-spin asymmetries $A_{\rm N}$, obtained from scattering one transversely polarized proton off an unpolarized one. It was found a long time ago in fixed-target experiments [1, 2] that such $p^{\uparrow}p$ collisions can yield strongly asymmetric distributions of hadrons in the final state. The most famous examples are the sizable ($\mathcal{O}(10\%)$) $A_{\rm N}$ found in the process $p\,p^{\uparrow} \to \pi\,X$, which express the fact that the produced pions have a preference to go to a particular side of the plane spanned by the proton beam direction and the transverse spin direction. Recently, the STAR collaboration at RHIC has found that such large asymmetries persist even at collider energies [3]. It is fair to say that to date the asymmetries have defied a full understanding at the quark-gluon level in QCD. One reason for this is that in QCD $A_{\rm N}$ for a single-inclusive reaction is power-suppressed as $1/p^{\perp}$ in the hard scale given by the transverse momentum $p^{\perp}$ of the pion. This makes the formalism for describing the asymmetries rather complicated, compared to more standard leading-power observables in perturbative QCD. An attempt of an explanation for the observed $A_{\rm N}$ has been given within a formalism [4] that systematically treats the power-suppression of $A_{\rm N}$ in terms of higher-twist parton correlation functions. Alternatively, it has been proposed [5, 6, 7, 8] that the dependence of parton distributions and fragmentations functions on a small "intrinsic" transverse momentum $k^{\perp}$ could be responsible for the asymmetries, through the interplay with the partonic elementary cross sections that are functions steeply falling with $p^{\perp}$. $A_{\rm N}$ is generated from the $k^{\perp}$-odd parts of the partonic scatterings, which acquire an additional factor $1/p^{\perp}$, making the mechanism again effectively higher twist. Measurements of just $A_{\rm N}$ in $p\,p^{\uparrow} \to \pi\,X$ will not be sufficient to disentangle all these effects, and it has also been shown recently that for this power-suppressed observable the mechanisms are not all independent of one another [9].

There is, however, a class of observables for which the $k^{\perp}$-dependent distributions or fragmentation functions alone are relevant, and may actually lead to leading-power effects. These are observables directly sensitive to a small measured transverse momentum. In spin physics, the most well-known example in this class was given by Collins [6]. He proposed to consider the single-transverse spin asymmetry in semi-inclusive deeply-inelastic

*Electronic address: dboer@nat.vu.nl
†Electronic address: vogelsan@quark.phy.bnl.gov

scattering (SIDIS), $e\,\vec{p} \to e'\,\pi\,X$, where the pion is detected out of the scattering plane. This asymmetry may receive contributions from the $k^\perp$ effects mentioned above: from the transverse momentum of the pion relative to its quark progenitor (the so-called Collins effect [6]), or from the intrinsic $k^\perp$ of a parton in the initial proton (referred to as Sivers mechanism [5]). Here we propose a new observable sensitive to the latter effect.

More precisely, the Sivers effect is a correlation between the direction of the transverse spin of the proton and the transverse momentum direction of an unpolarized parton inside the proton [5]. The Sivers effect in the process $p\,p^\uparrow \to \pi\,X$ has first been analyzed in detail by Anselmino *et al.* [7], who extracted the Sivers functions for valence quarks from a fit to the data under the assumption that the asymmetry is solely due to this effect. Subsequently, the single spin asymmetry for the Drell-Yan process, which is another process that belongs to the class of "leading-power" observables mentioned above, was predicted [10].

Following in part the notation of Ref. [10], the number density of a parton $f = u, \bar{u}, \ldots, g$ inside a proton with transverse polarization $\boldsymbol{S}_T$ and three-momentum $\boldsymbol{P}$, is parameterized as

$$\hat{f}(x, \boldsymbol{k}^\perp, \boldsymbol{S}_T) = f(x, k^\perp) + \frac{1}{2}\Delta^N f(x, k^\perp) \frac{\boldsymbol{S}_T \cdot (\boldsymbol{P} \times \boldsymbol{k}^\perp)}{|\boldsymbol{S}_T|\,|\boldsymbol{P}|\,|\boldsymbol{k}^\perp|}, \tag{1}$$

where $\boldsymbol{k}^\perp$ is the quark's transverse momentum, with $k^\perp = |\boldsymbol{k}^\perp|$. The function $f(x, k^\perp)$ is the unpolarized parton distribution, and $\Delta^N f$ denotes the Sivers function. As one can see, the correlation proposed by Sivers corresponds to a time-reversal odd triple product $\boldsymbol{S}_T \cdot (\boldsymbol{P} \times \boldsymbol{k}^\perp)$. Since for stable initial hadrons no strong interaction phases are expected, until recently it was widely believed that the Sivers functions had to vanish identically. It was then discovered [9, 11, 12, 13], however, that the time-reversal symmetry argument against the Sivers functions is invalidated by the presence of the Wilson lines in the operators defining the parton densities. These are required by gauge invariance and, as under time reversal future-pointing Wilson lines turn into past-pointing ones, the time reversal properties of the Sivers functions are non-trivial and permit them to be non-vanishing. It is intriguing that the possibility of a non-vanishing Sivers function emerges solely from the Wilson lines in QCD. Another aspect to the physics importance of the Sivers function is the fact that it arises as an interference of wave functions with angular momenta $J_z = \pm 1/2$ and hence contains information on parton orbital angular momentum [11, 14, 15].

In this paper we will discuss a single-spin asymmetry in $p\,p$ scattering that also belongs to the class of "leading-power" observables. The reaction we will consider is the inclusive production of jet pairs, $p\,p^\uparrow \to \mathrm{jet}_1\,\mathrm{jet}_2\,X$, for which the two jets are nearly back-to-back in azimuthal angle. This requirement makes the jet pairs sensitive to a small measured transverse momentum, and hence allows the single-spin asymmetry for the process to be of leading power. The asymmetry A_{N} for this process should give direct access to the Sivers function. Actually, in contrast to SIDIS and to $p^\uparrow p \to \pi X$, the observable we propose here has the feature that it is rather sensitive to the nonvalence contributions to the Sivers effect, in particular the gluon Sivers function. The latter has not been considered so far in any phenomenological asymmetry study. Its precise definition has been given by Mulders and Rodrigues [16]. They define a gluon correlation function (in the light-cone ($A^+ = 0$) gauge)

$$M\Gamma^{ij}(x, \boldsymbol{k}_T) \equiv \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{ik\cdot\xi} \langle P, S|F^{+i}(0) F^{+j}(\xi)|P, S\rangle|_{\xi^+=0}\ , \tag{2}$$

which can be parameterized in terms of gluon distribution functions. In particular, one has

$$M\Gamma_{ij}(x, \boldsymbol{k}_T) g_T^{ij} = -xP^+ \left[G(x, \boldsymbol{k}_T^2) + \frac{\epsilon_T^{k_T S_T}}{M} G_T(x, \boldsymbol{k}_T^2) \right]\ , \tag{3}$$

where $G(x, \boldsymbol{k}_T^2)$ is the unpolarized transverse momentum dependent gluon distribution inside an unpolarized hadron. Eqs. (2) and (3) can be viewed as an explicit definition of Eq. (1) in the gluon sector, i.e., G_T corresponds to $\Delta^N g(x, k^\perp)$, up to a $k^\perp$ dependent normalization factor.

As the above shows, the inclusion of transverse momentum dependent parton distributions is necessary when discussing Sivers effect asymmetries. This extension of the ordinary parton distributions that are functions of the lightcone momentum fractions only is not straightforward from a theoretical point of view. Factorization theorems involving $k^\perp$ dependence are generally harder to derive. Also, Sudakov suppression effects become relevant. As is well-known from unpolarized hadron-hadron collisions, the average transverse momenta of pair

final states, naively associated with originating from intrinsic transverse momentum, are energy ($\sqrt{s}$) dependent and can reach several GeV at collider energies [17, 18]. Clearly, such large average transverse momenta are not to be attributed to intrinsic transverse momenta alone, but mostly to the transverse momentum broadening due to (soft) gluon emissions. In our study we will therefore include such Sudakov effects, albeit within a somewhat simplified treatment. We will use experimental data to obtain estimates for the average transverse momenta of initial and radiated partons in unpolarized hadron-hadron collisions, which we will subsequently use to obtain estimates for the Sivers effect asymmetry in the azimuthal angular distribution of jets with respect to opposite side jets. We will do this without and with inclusion of Sudakov factors; one has to keep in mind that in the first case the average "intrinsic" transverse momenta we will find will effectively contain a significant perturbative (Sudakov) component. It will be instructive to compare the results of the two analyses.

Our study is also motivated by the favorable experimental situation at RHIC, where the STAR collaboration has recently presented data [19] for a closely related back-to-back reaction. In the next section we will discuss back-to-back correlations in the unpolarized case and compare to the STAR data. In Sec. III we will then address the spin asymmetry in $p\,p^{\uparrow} \to \mathrm{jet}_1\,\mathrm{jet}_2\,X$. Section IV presents our conclusions and a further discussion of some theoretical issues.

II. JET CORRELATIONS IN UNPOLARIZED HADRON COLLISIONS

We first consider the inclusive production of jet pairs in unpolarized proton-proton collisions, $p\,p \to \mathrm{jet}_1\,\mathrm{jet}_2\,X$. We take each of the jets to have large transverse momentum. This implies the presence of short-distance phenomena, which may be separated from long-distance ones. More precisely, the cross section for this process factorizes into convolutions of parton distribution functions with partonic hard-scattering cross sections that may be evaluated using QCD perturbation theory. To lowest order, the partonic subprocesses are the QCD two-parton to two-parton scatterings. If the cross section observable is defined in such a way that it is insensitive to transverse momenta of the initial partons or to particles radiated at small transverse momentum, the factorization is the standard "collinear" one, and the convolutions are simply in terms of (light-cone) momentum fractions. The observable we are interested in is slightly more involved. We choose the two jets to be almost back-to-back when projected into the plane perpendicular to the direction of the beams, which is equivalent to the jets being separated by nearly $\Delta\phi \equiv \phi_{j_2} - \phi_{j_1} = \pi$ in azimuth. Such a configuration directly corresponds to an observed small transverse momentum of the jet pair. In the case of two-by-two scattering of collinear initial partons, the jets are exactly back-to-back in azimuth. Deviations from this may result from additional partons being radiated into the final state. If the jets have fairly large separation in azimuthal angle, the dominant contribution to the cross section will come from a single additional parton radiated into the final-state against which the two jets recoil. Closer to $\Delta\phi = \pi$, radiation is suppressed, and Sudakov effects become relevant[1]. Intrinsic transverse momenta of the initial partons may become important as well.

In such nearly back-to-back situations, factorization is not necessarily lost; rather, a factorization theorem now needs to be formulated in terms of parton distributions depending on light-cone momentum fraction *and* transverse momentum. Factorization theorems of this type have been discussed for the simpler process $e^+\,e^- \to A\,B\,X$, where A and B are two hadrons almost back-to-back [20, 21], and for Drell-Yan type processes [22]. For factorization to occur it is essential that transverse momenta of initial or radiated partons are linked to the observed small transverse momentum only kinematically, that is, by momentum conservation, but are neglected in the hard scattering. For $p\,p \to \mathrm{jet}_1\,\mathrm{jet}_2\,X$, factorization at small measured transverse momentum of the pair has to our knowledge not yet been proven explicitly, but here we will assume that it falls in the class discussed in Ref. [22].

In the following we first consider only intrinsic transverse momentum of the initial partons and neglect perturbative radiation of particles into the final state and Sudakov effects. The latter will generally be very relevant and we will include them afterwards to leading logarithm for the purpose of estimating them. A treatment beyond

[1] Hard three (or more) jet configurations may also contribute near $\Delta\phi = \pi$, if the additional hard parton happens to be almost in one plane with the two jets. Such situations are expected to be relatively rare because they are not associated with singular behavior of the perturbative cross section in the azimuthal back-to-back region. We ignore them in our analysis.

leading logarithm is fairly complicated for $p\,p \to \mathrm{jet}_1\,\mathrm{jet}_2\,X$ and not within the scope of this work. Our treatment follows the same strategy as applied in Refs. [8, 23].

In the absence of additional partons being radiated into the final state, momentum conservation implies that the sum of the transverse momenta of the two jets is equal to the sum of the transverse momenta of the initial state partons. To be more explicit, we expect the cross section to be proportional to

$$\mathcal{U} = \int d^2k_1^\perp d^2k_2^\perp\ \delta^2\left(\boldsymbol{k}_1^\perp + \boldsymbol{k}_2^\perp - \boldsymbol{P}_{j_1}^\perp - \boldsymbol{P}_{j_2}^\perp\right)\ f_1(k_1^\perp)\ f_2(k_2^\perp)\ , \tag{4}$$

where $\boldsymbol{P}_{j_1}^\perp$ and $\boldsymbol{P}_{j_2}^\perp$ are the transverse momenta of the jets, and the f_i are the transverse momentum distributions of the initial partons. In general, the f_i will also depend on the parton lightcone momentum fractions. The factor $\mathcal{U}$ may be thought of as a smeared-out $\delta^2\left(\boldsymbol{P}_{j_1}^\perp + \boldsymbol{P}_{j_2}^\perp\right)$ representing the standard transverse-momentum conservation for collinear partons.

Since the distributions f_i are not known, we will assume Gaussian transverse momentum dependence for simplicity:

$$f_i(k_i^\perp) = \frac{\mathrm{e}^{-\left(k_i^\perp\right)^2/\langle k_i^{\perp\,2}\rangle}}{\pi\langle k_i^{\perp\,2}\rangle}\ . \tag{5}$$

Moreover, we will assume that the average transverse momentum squared is the same for all partons in the proton and independent of x, i.e. $\langle k_i^{\perp\,2}\rangle \equiv \langle k^{\perp\,2}\rangle$ for $i = 1, 2$. These simplifications may all be improved upon at a later stage, when there are data requiring a more sophisticated treatment, but here we will focus on the proof of principle rather than on making an accurate quantitative prediction. The assumption of Gaussians is for convenience and sufficient for our purpose. One obtains

$$\mathcal{U} = \frac{\mathrm{e}^{-(r^\perp)^2/\left(\langle k_1^{\perp\,2}\rangle + \langle k_2^{\perp\,2}\rangle\right)}}{\pi\left(\langle k_1^{\perp\,2}\rangle + \langle k_2^{\perp\,2}\rangle\right)}\ , \tag{6}$$

where $\boldsymbol{r}^\perp = \boldsymbol{P}_{j_1}^\perp + \boldsymbol{P}_{j_2}^\perp$. One has

$$(\boldsymbol{r}^\perp)^2 = P_{j_1}^{\perp\,2} + P_{j_2}^{\perp\,2} + 2P_{j_1}^\perp P_{j_2}^\perp \cos(\Delta\phi)\ , \tag{7}$$

where $\Delta\phi$ is the separation of the two jets in azimuth. Hence, one finds that

$$\mathcal{U} \propto e^{-K_\mathcal{U}\cos(\Delta\phi)}, \tag{8}$$

where $K_\mathcal{U} = P_{j_1}^\perp P_{j_2}^\perp/\langle k^{\perp\,2}\rangle$. This implies that $\mathcal{U}$ is peaked around $\Delta\phi = \pi$ as expected. Expansion for small $\delta\phi \equiv \Delta\phi - \pi$ shows a Gaussian behavior near the peak.

Correlations in $\Delta\phi$ for dijets have been studied in [24]. Measurements of $\Delta\phi$ distributions have also been performed both at the ISR [25], and in the fixed-target experiment E706 [17, 18], albeit not for two-jet correlations, but rather for pairs of nearly back-to-back leading hadrons, usually pions. Recently, the STAR collaboration [19] at RHIC (BNL) has presented precise data on hadron $\Delta\phi$ correlations from unpolarized proton-proton collisions at $\sqrt{S} = 200$ GeV (and from heavy ion collisions). Our study of spin effects in back-to-back reactions in the next section will be tailored to $p\,p^\uparrow$ collisions at $\sqrt{S} = 200$ GeV at RHIC, so we will compare our approach to the STAR $\Delta\phi$ distribution data [19], displayed for "same-sign" hadrons in $-\pi/2 \le \delta\phi \le \pi/2$ in Fig. 1. In the STAR analysis, the first ("trigger") hadron was required to have 4 GeV $\le P_{h_1}^\perp \le 6$ GeV, and the recoiling hadron 2 GeV $\le P_{h_2}^\perp \le P_{h_1}^\perp$. The pseudorapidities of both hadrons were within $|\eta_{h_{1,2}}| \le 0.7$. As can be seen from Fig. 1, the peak at $\delta\phi = \Delta\phi - \pi = 0$ is clearly pronounced and appears consistent with a Gaussian behavior in $\delta\phi$. One also notices that the distribution does not decrease to zero at large $\delta\phi$, which could be indicative of the perturbative tail corresponding to hard-gluon emission.

The above considerations for dijet production can be modified to apply to dihadron production. The first modification is that one needs to take into account fragmentation functions describing the probability with which

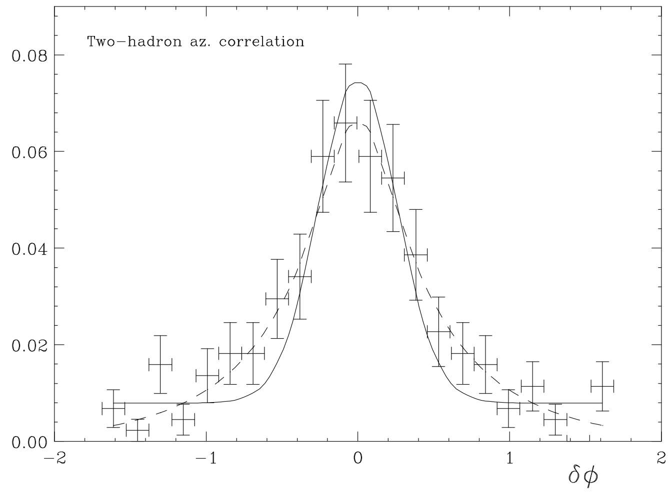

FIG. 1: Two-hadron azimuthal correlation in the back-to-back region. See Ref. [19] for details on the experimental definition of the correlation function. The data are from [19], and the solid curve is obtained using the distribution (8) and the widths in Eq. (13), fitting the overall normalization and an additive constant. The dashed line shows the result of a fit to the data with inclusion of leading-logarithmic Sudakov effects.

a final-state parton emerging from the hard scattering will yield the observed hadron. The hadron will take a light-cone momentum fraction z of the parton momentum p, i.e. in particular for a collinear fragmentation process:

$$\boldsymbol{P}_{h_1}^{\perp} = z_1 \boldsymbol{p}_1^{\perp} \ , \quad \boldsymbol{P}_{h_2}^{\perp} = z_2 \boldsymbol{p}_2^{\perp} \tag{9}$$

for the transverse components, and hence $\boldsymbol{r}^{\perp} = \boldsymbol{P}_{h_1}^{\perp}/z_1 + \boldsymbol{P}_{h_2}^{\perp}/z_2$ in the formulas above. For our estimate we will use in the following only average values for the hadron transverse momenta over the experimental bins: $\langle P_{h_1}^{\perp}\rangle \approx 4.5$ GeV, $\langle P_{h_2}^{\perp}\rangle \approx 2.5$ GeV. We have then determined the corresponding average z_i in the framework of a leading-order calculation of the unpolarized dihadron cross section in $p\,p$ scattering at RHIC energy $\sqrt{S} = 200$ GeV and find $\langle z_1\rangle \approx 0.45$, $\langle z_2\rangle \approx 0.25$. Here we have used the CTEQ-5 [26] set of parton distribution functions and the fragmentation functions of [27], both at leading order.

Additionally, there will be a transverse-momentum smearing also in the final state, that is, the observed hadron may be produced at some small transverse momentum relative to the parent parton [28], implying

$$\boldsymbol{P}_{h_1}^{\perp} = z_1 \boldsymbol{p}_1^{\perp} + \hat{\boldsymbol{k}}_1^{\perp} \ , \quad \boldsymbol{P}_{h_2}^{\perp} = z_2 \boldsymbol{p}_2^{\perp} + \hat{\boldsymbol{k}}_2^{\perp} \ , \tag{10}$$

with $\boldsymbol{p}_i^{\perp} \cdot \hat{\boldsymbol{k}}_i^{\perp} = 0$. This will also have an influence on the $\delta\phi$ distribution of the two produced hadrons. We estimate this effect by replacing $\langle k_1^{\perp\,2}\rangle + \langle k_2^{\perp\,2}\rangle$ in Eq. (8) by $\langle k_1^{\perp\,2}\rangle + \langle k_2^{\perp\,2}\rangle + \langle \hat{k}_1^{\perp\,2}\rangle/\langle z_1\rangle^2 + \langle \hat{k}_2^{\perp\,2}\rangle/\langle z_2\rangle^2$, where $\langle \hat{k}_i^{\perp\,2}\rangle$ is the average transverse momentum broadening squared in fragmentation. We could improve this estimate by taking into account that for a final-state particle emitted at some angle only a certain projection of $\hat{\boldsymbol{k}}^{\perp}$ is relevant for the $\delta\phi$ distribution. This modifies the functional form of the distribution in $\delta\phi$. However, we found this effect to be rather insignificant for the $\delta\phi$ and the widths we consider below.

Our next goal is to obtain an estimate of the average $\langle k_i^{\perp\,2}\rangle$ at RHIC energy $\sqrt{S} = 200$ GeV from a comparison to experimental data. For this, Ref. [29] is particularly useful, where an analysis of a variety of data from fixed-target and collider experiments on transverse momenta of dimuon, diphoton, and dijet (or dihadron) pairs was performed. If one neglects radiative effects, such pair transverse momenta are directly related to intrinsic transverse momenta. The results of [29] show that the pair transverse momenta increase with center-of-mass energy. Also, they are consistently larger for dihadron pairs than for diphoton or dimuon pairs. This may be understood from the presence of $k^{\perp}$ smearing in fragmentation. Additionally, in contrast to diphotons or dimuons, dihadron cross sections are dominated by scatterings of initial gluons, which may have a somewhat larger $k^{\perp}$ broadening. As mentioned above, we neglect this effect. From the results shown in [29] we then

estimate for dihadrons at $\sqrt{S}=200$ GeV

$$\langle k_1^{\perp\,2}\rangle + \langle k_2^{\perp\,2}\rangle + \langle \hat{k}_1^{\perp\,2}\rangle/\langle z_1\rangle^2 + \langle \hat{k}_2^{\perp\,2}\rangle/\langle z_2\rangle^2 \;\approx\; 15\ \mathrm{GeV}^2\ , \tag{11}$$

and for non-fragmentation final states

$$\langle k_1^{\perp\,2}\rangle + \langle k_2^{\perp\,2}\rangle \;\approx\; 9\ \mathrm{GeV}^2\ . \tag{12}$$

From this, we estimate, assuming $\langle k_1^{\perp\,2}\rangle = \langle k_2^{\perp\,2}\rangle$ and $\langle \hat{k}_1^{\perp\,2}\rangle = \langle \hat{k}_2^{\perp\,2}\rangle$:

$$\sqrt{\langle k_i^{\perp\,2}\rangle} \approx 2\ \mathrm{GeV}\ , \qquad \sqrt{\langle \hat{k}_i^{\perp\,2}\rangle} \approx 0.5\ \mathrm{GeV}\ . \tag{13}$$

Our value for the width for the initial-state broadening is larger than that found from studies of single-particle inclusive cross sections at the lower fixed-target energies in [10]. The curve in Fig. 1 shows our result for the $\delta\phi$ distribution based on the widths in Eq. (13). We have fitted the overall normalization of the curve to the data, and we have also allowed an additive constant to the $\delta\phi$ distribution in this fit, in order to account for the perturbative tail at larger $|\delta\phi|$. The resulting curve gives a fair description of the data, even though the data appear to prefer a somewhat larger width of the peak.

The fairly large size of $\sqrt{\langle k_i^{\perp\,2}\rangle}$ in Eq. (13) (as compared to typical hadronic mass scales) again indicates that there are significant perturbative effects that should be taken into account in a more thorough analysis. As we mentioned earlier, Sudakov effects, related to multi-soft-gluon emission, are expected to be particularly relevant. Near $\delta\phi=0$, gluon radiation is kinematically inhibited, and the standard cancelations of infrared singularities between virtual and real diagrams lead to large logarithmic remainders in the partonic hard-scattering cross sections. For the $\delta\phi$ distribution, these have the form $\alpha_s^k \ln^{2k-m}(\delta\phi)/\delta\phi$ in kth order of perturbation theory, with $1\le m\le 2k$ or, more generally, for the $\boldsymbol{r}^\perp$ distribution they are of the form $\alpha_s^k \ln^{2k-m}(\hat{s}/|\boldsymbol{r}^\perp|^2)/|\boldsymbol{r}^\perp|^2$ [30, 31], where $\sqrt{\hat{s}}$ is the partonic center-of-mass energy. It is possible to resum these logarithmic contributions to all orders in α_s. For the leading logarithms ($m=1$), this was achieved a long time ago [30, 32, 33]. Recent progress in the resummation for $p\,p\to A\,B\,X$ at next-to-leading logarithmic level was reported in [34]. Here we provide an estimate of the Sudakov effects by taking into account the tower of leading logarithms.

Applying the derivation of [30] to the process $p\,p\to \mathrm{jet}_1\,\mathrm{jet}_2\,X$, the resummation of independent soft gluon emissions to leading double logarithmic order leads to the following distribution in $\boldsymbol{r}^\perp$:

$$\mathcal{U}^{\,\mathrm{Sud}} \equiv \frac{1}{\sigma_0}\frac{d\sigma}{d^2r^\perp} = \int_0^\infty \frac{db^2}{4\pi}\, J_0(br^\perp)\, \exp\left[-\frac{\alpha_s}{\pi}\, C\, \ln^2(b\sqrt{\hat{s}})\right]\, \tilde{f}_1(b^2)\tilde{f}_2(b^2), \tag{14}$$

where σ_0 is the lowest-order cross section integrated over all $r^\perp$, and C is the sum of the color charges for the external legs in the partonic hard scattering, i.e., for subprocesses involving quarks and antiquarks only one has $C=4C_F=16/3$, for processes with two quarks and two gluons $C=2(C_A+C_F)=26/3$, and for $gg\to gg$, $C=4C_A=12$. All these channels are relevant, because of the competition between the magnitude of the parton distributions and the magnitude of the Sudakov suppression. Finally, in Eq. (14)

$$\tilde{f}_i(b^2) = \int d^2k_i^\perp\, e^{i\boldsymbol{b}\cdot\boldsymbol{k}_i^\perp}\, f_i(k_i^\perp) = e^{-b^2\langle k_i^{\perp\,2}\rangle/4}\ , \tag{15}$$

where the last equality follows for our Gaussian $k^\perp$ distributions of Eq. (5). In lowest order in α_s, i.e. setting $\alpha_s=0$ in the Sudakov exponent, Eq. (14) reduces to $\mathcal{U}$ of Eq. (4).

We have used the Sudakov improved $\mathcal{U}^{\,\mathrm{Sud}}$ of Eq. (14) in a fit of the value for $\langle k^{\perp\,2}\rangle$ to the STAR data. We find that inclusion of the leading logarithms leads to a markedly better agreement with the data, demonstrated by the dashed line in Fig. 1, and to a reasonably small value of $\sqrt{\langle k^{\perp\,2}\rangle}\approx 0.9$ GeV, which is closer to a typical hadronic mass scale and to the one obtained in [10].

We note that our treatment of the soft-gluon resummation for the $\delta\phi$ distribution somewhat differs from that developed in [33, 35], where the resummation was performed in terms of a one-dimensional integral transform. We found our approach, which is based on a two-dimensional impact parameter $\boldsymbol{b}$ [30], to be numerically very similar to the one of [33] near $\delta\phi=0$, but to be better applicable out to larger $\delta\phi$. A more complete study of

the Sudakov effects would start from the well-known [31] full form of the Sudakov exponent, given in terms of an integral over gluon transverse momentum. This form reduces to ours in Eq. (14) if the running of the strong coupling is neglected. For a running coupling, the Sudakov exponent becomes sensitive to the strong-coupling regime, since α_s is also probed at scales near $\Lambda_{\rm QCD}$. This will require the introduction of further non-perturbative contributions. It may also be important to determine next-to-leading logarithmic contributions to the exponent [34]. The issue of matching to a fixed (next-to-leading) order calculation [36] at larger $\delta\phi$ will then become relevant.

We finally note that, besides the peak around $\Delta\phi = \pi$, the two-hadron azimuthal correlation also shows a peak at $\Delta\phi = 0$, corresponding to the two hadrons being in the same jet. The width of this peak should be primarily related to the "fragmentation part" $\langle \hat{k}_1^{\perp\,2}\rangle/\langle z_1\rangle^2 + \langle \hat{k}_2^{\perp\,2}\rangle/\langle z_2\rangle^2$ of the width in Eq. (11). One therefore expects the peak at $\Delta\phi = 0$ to be narrower than the one at $\Delta\phi = \pi$, which indeed it is for the STAR data. We have checked that a distribution of the form (8), with only fragmentation $k^\perp$ broadening as numerically given by Eqs. (11) and (12), fits the $\Delta\phi = 0$ peak well. We note that a detailed description of this region will require using two-hadron fragmentation functions as studied in [37, 38, 39]. In the following we will not consider further the same-side peak at $\Delta\phi = 0$.

III. JET CORRELATIONS IN $pp^\uparrow$ SCATTERING

We will now study two-jet correlations near $\Delta\phi = \pi$ in the case that one of the two incoming protons is polarized transversely to its momentum. As we discussed in the introduction, our motivation is that such correlations may offer access to the Sivers function. The basic idea is very simple. As follows from Eq. (1), the Sivers function represents a correlation of the form $\boldsymbol{S}_T\cdot(\boldsymbol{P}\times\boldsymbol{k}^\perp)$ between the transverse proton polarization vector, its momentum, and the transverse momentum of the parton relative to the proton direction. In other words, if there is a Sivers-type correlation then there will be a preference for partons to have a component of intrinsic transverse momentum to one side, perpendicular to both $\boldsymbol{S}_T$ and $\boldsymbol{P}$. Suppose now for simplicity that one observes a jet in the direction of the proton polarization vector, as shown in Fig. 2. A "left-right" imbalance in $\boldsymbol{k}^\perp$ of the parton will then affect the $\Delta\phi$ distribution of jets nearly opposite to the first jet and give the cross section an asymmetric piece around $\Delta\phi = \pi$. The spin asymmetry

$$A_{\rm N} \equiv \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow} \tag{16}$$

will extract this piece.

The denominator of this asymmetry will be, up to normalization, the function $\mathcal{U}$ discussed in Sec. II. We now define the y, z directions as given by the polarization and momentum, respectively, of the polarized proton. The numerator may then be found by considering Eq. (4) with an additional factor $k_1^{\perp\,x}$ in the integrand, corresponding to the Sivers correlation in the polarized proton:

$$\mathcal{P} = \int d^2k_1^\perp d^2k_2^\perp \; \frac{k_1^{\perp\,x}}{\sqrt{\langle \kappa_1^{\perp\,2}\rangle}} \; \delta^2\left(\boldsymbol{k}_1^\perp + \boldsymbol{k}_2^\perp - \boldsymbol{P}_{j_1}^\perp - \boldsymbol{P}_{j_2}^\perp\right) \; \bar{f}_1(k_1^\perp)\; f_2(k_2^\perp)\,, \tag{17}$$

where we have introduced the bar on f_1 to indicate that $\bar{f}_1$ is related to the transverse momentum dependent part of a Sivers function, with Gaussian width $\langle \kappa_1^{\perp\,2}\rangle$. The latter will in general be different from that in the unpolarized distribution (in fact, smaller to satisfy a positivity bound, see Ref. [10]). We have normalized $k_1^{\perp\,x}$ by $\sqrt{\langle \kappa_1^{\perp\,2}\rangle}$ instead of by $|\boldsymbol{k}_1^\perp|$ as Eq. (1) suggests. This follows the analysis of Ref. [10] (cf. Eq. (24) below) and takes care of the fact that for $\boldsymbol{k}_1^\perp = 0$ the Sivers effect should vanish.

In this way we obtain

$$\mathcal{P} = \frac{r^{\perp\,x}\sqrt{\langle \kappa_1^{\perp\,2}\rangle}}{\pi\left(\langle \kappa_1^{\perp\,2}\rangle + \langle k_2^{\perp\,2}\rangle\right)^2}\; \mathrm{e}^{-(r^\perp)^2/\left(\langle \kappa_1^{\perp\,2}\rangle + \langle k_2^{\perp\,2}\rangle\right)}\,, \tag{18}$$

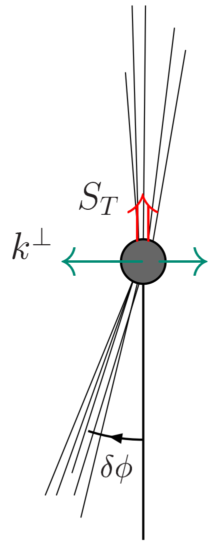

FIG. 2: Asymmetric jet correlation. The proton beams run perpendicular to the drawing.

or in terms of the jet transverse momenta and azimuthal angles (measured w.r.t. $\boldsymbol{S}_T$),

$$\mathcal{P} = \left(|\boldsymbol{P}_{j_1}^\perp|\sin\phi_{j_1} + |\boldsymbol{P}_{j_2}^\perp|\sin\phi_{j_2}\right) \frac{\sqrt{\langle\kappa_1^{\perp\,2}\rangle}}{\pi\left(\langle\kappa_1^{\perp\,2}\rangle + \langle k_2^{\perp\,2}\rangle\right)^2} e^{-\left[P_{j_1}^{\perp\,2} + P_{j_2}^{\perp\,2} + 2P_{j_1}^\perp P_{j_2}^\perp \cos(\Delta\phi)\right]/\left(\langle\kappa_1^{\perp\,2}\rangle + \langle k_2^{\perp\,2}\rangle\right)}\ . \tag{19}$$

As an example, for the case of $\phi_{j_1} = 0$, corresponding to our specific example displayed in Fig. 2, this yields

$$\mathcal{P} \propto \sin(\Delta\phi) e^{-K_{\mathcal{P}}\cos(\Delta\phi)}, \tag{20}$$

where $K_{\mathcal{P}} = 2P_{j_1}^\perp P_{j_2}^\perp/(\langle\kappa_1^{\perp\,2}\rangle + \langle k_2^{\perp\,2}\rangle)$.

Setting $\langle\kappa^{\perp\,2}\rangle = r\langle k^{\perp\,2}\rangle$, the resulting spin asymmetry is

$$A_{\rm N} = \frac{r^{\perp\,x}}{\sqrt{\langle k^{\perp\,2}\rangle}} \frac{2\sqrt{r}}{(1+r)^2} \mathrm{e}^{-\frac{1-r}{2(1+r)}(r^\perp)^2/\langle k^{\perp\,2}\rangle}\ , \tag{21}$$

where for the denominator of this asymmetry we have taken the function $\mathcal{U}$ given in Eq. (6) with the assumption $\langle k_i^{\perp\,2}\rangle \equiv \langle k^{\perp\,2}\rangle$ for $i = 1, 2$.

There are, however, several different partonic channels contributing to jet production in pp scattering. For each of these, there is a combination of parton distributions, with dependence on light-cone momentum fraction and intrinsic transverse momentum. Thus in general, there will be a weighted sum of $k^\perp$ dependent functions in the numerator and the denominator of the asymmetry, that is,

$$A_{\rm N} = \frac{\sum_{f_1,f_2}\int d^2\boldsymbol{k}_1^\perp d^2\boldsymbol{k}_2^\perp \delta^2\left(\boldsymbol{k}_1^\perp + \boldsymbol{k}_2^\perp - \boldsymbol{r}^\perp\right) \frac{k_1^{\perp\,x}}{k_1^\perp}\,\Delta^N f_1(x_1,k_1^\perp)\otimes f_2(x_2,k_2^\perp)\otimes\hat\sigma_{f_1f_2}(P_{j_1}^\perp,P_{j_2}^\perp,\eta_{j_1},\eta_{j_2})}{\sum_{f_1,f_2}\int d^2\boldsymbol{k}_1^\perp d^2\boldsymbol{k}_2^\perp \delta^2\left(\boldsymbol{k}_1^\perp + \boldsymbol{k}_2^\perp - \boldsymbol{r}^\perp\right)\ f_1(x_1,k_1^\perp)\otimes f_2(x_2,k_2^\perp)\otimes\hat\sigma_{f_1f_2}(P_{j_1}^\perp,P_{j_2}^\perp,\eta_{j_1},\eta_{j_2})}\ . \tag{22}$$

Here, the $\Delta^N f_1(x_1,k_1^\perp)$ are the Sivers functions as introduced in Eq. (1). The convolutions $\otimes$ are over light-cone momentum fractions only. We note that since the Sivers functions correspond to distributions of unpolarized partons, the hard-scattering cross sections $\hat\sigma_{f_1f_2}$ are the usual unpolarized ones in both the numerator and the denominator of the asymmetry. The pseudorapidities of the jets are denoted by η_{j_1} and η_{j_2}. The hard scattering functions depend only on large scales, that is, on $P_{j_1}^\perp$ and $P_{j_1}^\perp$. Therefore, any dependence on $\boldsymbol{k}_{1,2}^\perp$ is neglected in the $\hat\sigma_{f_1f_2}$; in other words, one considers the first term in a collinear expansion.

One can see that a simple result such as Eq. (21) will only emerge if the x and $\boldsymbol{k}^\perp$ dependences in all functions factorize from each other, if all distributions in the numerator and, separately, in the denominator depend on

$\boldsymbol{k}^\perp$ in the same way, and if the x-dependence for each Sivers function is identical to that of the corresponding unpolarized one.

We will now give simple estimates for the possible size of the spin asymmetry $A_{\rm N}$. For this purpose we will need a model for the dependence of the parton distributions on the light-cone momentum fraction x as well as on transverse momentum. We will assume that the x and $\boldsymbol{k}^\perp$ dependences may indeed be separated for each function. For the unpolarized densities we write:

$$f(x,k^\perp) = f(x)\,\frac{1}{\pi\langle k^{\perp\,2}\rangle} e^{-(k^\perp)^2/\langle k^{\perp\,2}\rangle}\;, \tag{23}$$

where the $f(x)$ are the usual unpolarized light-cone distributions for flavors $f = u,\bar{u},\ldots,g$, for which we again use the CTEQ-5 leading order set [26]. For the moment, we will neglect Sudakov effects, so we will use the value $\sqrt{\langle k^{\perp\,2}\rangle} = 2$ GeV we found in the previous section for the initial-state broadening without resummation of Sudakov logarithms.

The parameterizations for the Sivers function we will use are taken from [10], where they were determined from comparisons to data of Ref. [1] on inclusive single spin asymmetries for $p^\uparrow p \to \pi X$:

$$\Delta^N f(x,k^\perp) = 2\mathcal{N}_f(x)\, f(x)\,\frac{1}{\pi\langle k^{\perp\,2}\rangle}\sqrt{2e(1-r)}\;\frac{k^\perp}{\sqrt{\langle \kappa^{\perp\,2}\rangle}}\, e^{-(k^\perp)^2/\langle \kappa^{\perp\,2}\rangle}\;. \tag{24}$$

We have chosen this parameterization and the accompanying fit values of Ref. [10] since these are the only ones available so far. The question of universality (i.e. process independence) of the Sivers function we leave as an unresolved issue. For a further discussion of this point see Sec. IV.

As before, $r = \langle \kappa^{\perp\,2}\rangle/\langle k^{\perp\,2}\rangle$, and we use $r = 0.7$, in accordance with the fit of Ref. [10]. This value allows for a good fit to the E704 data [1] and also to Λ polarization data, as discussed in Ref. [40]. The $\mathcal{N}_f(x)$ are x-dependent normalizations, defined in [10] as

$$\mathcal{N}_f(x) = N_f x^{a_f}(1-x)^{b_f}\,\frac{(a_f+b_f)^{(a_f+b_f)}}{{a_f}^{a_f}{b_f}^{b_f}}\;. \tag{25}$$

In Ref. [10], only the valence u and d Sivers functions were taken into account, since the data of [1] are in the forward region of the polarized proton, corresponding to large momentum fractions in its parton distribution functions. For u and d, the parameters extracted from comparison to the data read [10]:

$$\begin{aligned} &N_u = 0.5,\; a_u = 2.0,\; b_u = 0.3\;, \\ &N_d = -1.0,\; a_d = 1.5,\; b_d = 0.2\;. \end{aligned} \tag{26}$$

For the sea quarks we assume relations identical to (25), with $\mathcal{N}_{\bar{u}}(x) = \mathcal{N}_u(x)$, $\mathcal{N}_{\bar{d},\bar{s}}(x) = \mathcal{N}_d(x)$ for simplicity. The details of these choices are not crucial.

The size of the asymmetry is, however, very sensitive to the gluon Sivers function. The reason for this is that in the kinematic regime we will investigate here, $\sqrt{s} = 200$ GeV, $P_j^\perp \sim 10$ GeV, contributions from gluon-gluon and quark-gluon scattering are most important, because of their large partonic cross sections and because the parton momentum fractions become as low as $x_{1,2} \sim 0.05$. There is so far no experimental information on the gluon Sivers function. It may be possible to obtain estimates within models of nucleon structure. This could be an interesting topic for future studies, but is beyond the scope of the present paper. To illustrate the dependence of the asymmetry $A_{\rm N}$ on the gluon Sivers function, we will simply present results for four distinct cases:

(i) $\mathcal{N}_g(x) = \left(\mathcal{N}_u(x) + \mathcal{N}_d(x)\right)/2$,

(ii) $\mathcal{N}_g(x) = 0$,

(iii) $\mathcal{N}_g(x) = \mathcal{N}_d(x)$,

(iv) $\mathcal{N}_g(x) = \mathcal{N}_d(x)$, but with $\sqrt{\langle k^{\perp\,2}\rangle} = 2.5$ GeV for gluons.

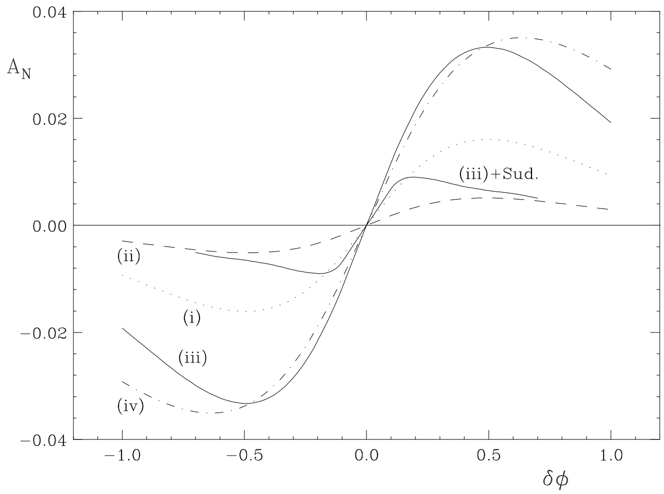

FIG. 3: Predictions for the spin asymmetry $A_{\rm N}$ for back-to-back dijet production at RHIC ($\sqrt{s} = 200$ GeV), for various different models (i)-(iv) for the gluon Sivers function (see text). The solid line marked as "(iii)+Sud." shows the impact of leading logarithmic Sudakov effects on the asymmetry for model (iii).

These choices represent a very small (scenario ii), an average (with respect to the functions for quarks, scenario i), and two somewhat larger (scenarios iii and iv) gluon Sivers functions. Other choices are clearly possible, including yet larger functions. Choice (iv) is motivated by the notion that intrinsic $k^\perp$ smearing could be larger for gluons. We keep $r = \langle \kappa^{\perp\,2}\rangle / \langle k^{\perp\,2}\rangle = 0.7$ in all cases, although this choice does not result from the fit of Ref. [10], where a gluon Sivers function was not included. We have found that a change in r for the gluons of 10 % leads to a change of about 6 % in the asymmetry at its peak.

The resulting asymmetries, as functions of $\delta\phi$, are shown in Fig. 3, at RHIC's $\sqrt{s} = 200$ GeV. For simplicity, we have chosen $\phi_{j_1} = 0$. We have taken into account jets with pseudorapidities $|\eta_{j_{1,2}}| \leq 1$ (as suitable for the STAR detector) and 8 GeV $\leq P^\perp_{j_{1,2}} \leq 12$ GeV. The strong sensitivity to the gluon Sivers function is evident from Fig. 3. One can see that sizable asymmetries are by all means possible. In fact, the asymmetry may easily be even much larger ($> 10\%$) if the gluon Sivers function is favorably close to the unpolarized gluon density. We expect asymmetries of $\sim 1\%$ to be easily measurable at RHIC. A typical value for the statistical uncertainty of such measurements may be estimated as [41]

$$\delta A_{\rm N} \approx \frac{1}{P\sqrt{\sigma \mathcal{L}}} \,, \tag{27}$$

where P is the beam polarization, $\mathcal{L}$ the integrated luminosity, and σ the unpolarized dijet cross section integrated over the kinematical bin we are interested in. Using $P = 0.5$, a moderate $\mathcal{L} = 10/$pb, and estimating $\sigma = 6\cdot 10^6$ pb, we find $\delta A_{\rm N} \approx 2 \cdot 10^{-4}$. It will of course be important to understand systematical uncertainties at a similar level.

It is straightforward to determine the angle where the asymmetry has its maximum. If we choose $\phi_{j_1} = 0$ and define $\delta\phi = \phi_{j_2} - \pi$, then we find:

$$\cos(\delta\phi_{\rm max}) \approx 1 - \frac{\langle k^{\perp\,2}\rangle (1+r)}{2 P^\perp_{j_1} P^\perp_{j_2} (1-r)} \,, \tag{28}$$

which as expected is a function of the observed jet transverse momenta, and of $\langle \kappa^{\perp\,2}\rangle$ and $\langle k^{\perp\,2}\rangle$. For our parameters given above, this yields $\delta\phi_{\rm max} \approx 0.48$ (for scenarios i, ii and iii). The value of the asymmetry at this $\delta\phi_{\rm max}$ depends of course on the magnitude of the Sivers effect functions.

As in the previous section we will also estimate the effect of Sudakov factors by including soft gluon emissions at the leading double logarithmic level. Their effect on the denominator $\mathcal{U}$ of the asymmetry has been described in Eq. (14). A difference is now that for jet pairs (unlike the case of inclusive hadron pairs) only the initial-state

broadening plays a role [42]. This means that we will now have $C = 8/3$ for processes with only initial quarks and/or antiquarks, $C = 13/3$ for qg scattering, and $C = 6$ for a gg initial state. For the numerator of the asymmetry one has to replace in Eq. (14)

$$\tilde{f}_1(b^2) \to \int d^2k_1^\perp \, e^{i\boldsymbol{b}\cdot\boldsymbol{k}_1^\perp} \, \frac{k_1^{\perp\,x}}{\langle \kappa_1^{\perp\,2}\rangle} \, \bar{f}_1(k_1^\perp) = \frac{i}{2} \, b^x \, e^{-b^2\langle\kappa_i^{\perp\,2}\rangle/4} \equiv \frac{i}{2} \, b^x \tilde{\tilde{f}}_1(b^2) \; . \tag{29}$$

This leads to

$$\mathcal{P} = -\frac{r^{\perp\,x}}{r^\perp} \sqrt{\langle \kappa^{\perp\,2}\rangle} \int_0^\infty \frac{db^2}{4\pi} \, bJ_1(br^\perp) \, \exp\left[-\frac{\alpha_s}{\pi} \, C \, \ln^2(b\sqrt{\hat{s}})\right] \tilde{\tilde{f}}_1(b^2)\tilde{f}_2(b^2) \; . \tag{30}$$

The resulting spin asymmetry is also shown in Fig. 3, for the case (iii) above, i.e., $\mathcal{N}_g(x) = \mathcal{N}_d(x)$. For this curve we have now for consistency used the smaller value $\sqrt{\langle k^{\perp\,2}\rangle} \approx 0.9$ GeV that we extracted from our Sudakov analysis of the STAR data in the previous section. This leads to a shift of the peak of the distribution closer to $\delta\phi = 0$. More importantly, it is evident that the inclusion of the Sudakov factors leads to a considerable suppression of A_{N}. Of course, this does not rule out a sizable spin asymmetry in principle. As we mentioned earlier on, depending on the normalization of the gluon Sivers function, we could have a much larger A_{N} than given by models (i)-(iv). In this context, we would also like to remark that the spin asymmetry for the single-inclusive reaction $p\,p^\uparrow \to \pi\,X$ at moderately high $p^\perp$, from which in [10] the valence quark Sivers functions were extracted, is also sensitive to a small transverse momentum, and hence susceptible to Sudakov effects. The analysis of Ref. [10] did not include Sudakov factors, which means that any effects of Sudakov suppression have effectively been absorbed into the Sivers function itself. In that sense the curve in Fig. 3 may include Sudakov suppression more than once. A Sudakov improved analysis of the asymmetry in $p\,p^\uparrow \to \pi\,X$ would therefore be desirable.

To gain statistics in experiment, one will not just select events with $\phi_{j_1} \approx 0$ and vary ϕ_{j_2}; rather one will want to integrate over bins in ϕ_{j_1}. Here one has to take care not to wash out the asymmetry by simply integrating over all ϕ_{j_1}. The asymmetry will in general have the following dependence on ϕ_{j_1} and $\delta\phi$:

$$A_{\mathrm{N}}(\phi_{j_1}, \delta\phi) \propto \Big[|\boldsymbol{P}_{j_1}^\perp| \sin\phi_{j_1} - |\boldsymbol{P}_{j_2}^\perp| \left(\sin\phi_{j_1}\cos\delta\phi + \cos\phi_{j_1}\sin\delta\phi\right)\Big] \; \mathcal{A}(\cos(\delta\phi)). \tag{31}$$

One possibility is to select "jet 1" in the hemisphere of the "spin-up" direction, and to weight the asymmetry with $\cos(\phi_{j_1})$ over this hemisphere:

$$A_{\mathrm{N}}(\delta\phi) \equiv \int d\phi_{j_1} \; \cos(\phi_{j_1}) \, A_{\mathrm{N}}(\phi_{j_1}, \delta\phi) \propto \sin\delta\phi \, \mathcal{A}(\cos(\delta\phi)). \tag{32}$$

One has to keep in mind that this weighted asymmetry has a complicated dependence on the transverse momenta of the two jets, which differ per event. In general, it will be rather involved to extract the normalization and the width of the Sivers functions from such a weighted asymmetry, but the above is one of the ways to obtain a Sivers asymmetry that is dependent on only the angle $\delta\phi$. More generally, one could project out the full $r^{\perp\,x}$ azimuthal angular dependence, preferably for a fine binning in the transverse momenta of the jets.

If one were to consider leading hadrons instead of the jets, one would run into the problem that there could be additional effects generating a single spin asymmetry at leading power. In the fragmentation process the Collins effect could contribute [6], which is a correlation between the transverse spin of a fragmenting quark and the transverse momentum direction of the outgoing hadron relative to that quark. We do not, however, think that this mechanism will be very important here. First of all, the fragmenting quark would need to have inherited its spin from the transverse spin of the proton. This means that the transversity densities of the proton would be involved, and that the partonic cross sections would depend on transverse spin in the initial and final states. These cross sections are much smaller than the unpolarized ones we used in our study above [43]. In addition, a major difference between the Collins and Sivers effects is that the gluon Sivers function is allowed to be nonzero, whereas a gluonic Collins functions is forbidden by helicity conservation. We are therefore confident that studies of $p\,p^\uparrow \to h_1\,h_2\,X$ (with h_1, h_2 two hadrons almost back-to-back in azimuth) at RHIC, for example in the PHENIX experiment (where it would complement the Drell-Yan single spin asymmetry measurements), would also be useful for learning about the Sivers functions.

IV. CONCLUDING REMARKS

We have proposed an observable that could provide access to the Sivers effect: a single transverse-spin asymmetry in the distribution in relative azimuthal angle $\Delta\phi \approx \pi$ of jets in a dijet pair. Unlike the more customary single-spin asymmetries for single-inclusive final states in $p\,p^{\uparrow}$ scattering, this observable is not power-suppressed in a large energy scale. It also has the advantage that it will be directly (and only) sensitive to the Sivers functions, in contrast to $A_{\rm N}$ for the process $p\,p^{\uparrow} \to \pi\,X$ for which several different competing mechanisms could be at work. Using experimental information on the average transverse momentum of initial and radiated partons from dimuon, diphoton, dijet and dihadron production in hadron-hadron collisions and using results from the study of Ref. [10] on the Sivers effect in $p\,p^{\uparrow} \to \pi\,X$, we have presented estimates for this new observable. These indicate that the asymmetry could well be at the few percent level, which should make it experimentally accessible at RHIC.

Our further analysis revealed, however, that Sudakov effects lead to a significant suppression of the asymmetry. We stress that this does not necessarily mean that the asymmetry must be small. It turns out that the unknown gluon Sivers function mainly drives the size of the asymmetry. We know of no theoretical reason why this distribution function should be small. In any case, any sign in experiment of the asymmetry we propose will be definitive evidence for the Sivers effect. We also point out that the distribution in azimuthal angle between the jets is only one example of a variety of similar observables in $p\,p$ scattering. Other closely related examples, which deserve further attention and may be equally suited for experimental studies, are the total transverse momentum of the jet pair, the distribution in "$P_{\rm out}$" (the momentum of one jet out of the plane spanned by the beam axis and the other jet's transverse momentum) [33], and the distribution in "$P^{\perp}$-balance" $z = -\boldsymbol{P}_{j_1}^{\perp} \cdot \boldsymbol{P}_{j_2}^{\perp}/(P_{j_2}^{\perp})^2$ [17]. Any of these may be obtained from different projections in the two-dimensional transverse-momentum plane and may be predicted using our formulas above.

We close with a few comments on some theoretical issues, which we hope also provide directions for future work. As we mentioned earlier on, for observables that have a hard scale but additionally involve an observed small transverse momentum, factorization theorems are rather hard to establish. For our back-to-back dijet distribution, the issue of whether or not factorization occurs still remains to be investigated. For the case of nontrivial polarization effects, such as the Sivers or Collins effects, this is a particularly relevant issue since, unlike in the unpolarized case, the effect itself already relies on the presence of non-perturbative "intrinsic" transverse momentum: the parton distribution functions need to have an intrinsically nonperturbative dependence on the transverse momentum, arising as $k^{\perp}/M$ or $k^{\perp}/\langle k^{\perp\,2}\rangle$.

In case factorization does apply, another related complication is the apparent non-universality of the Sivers functions. When it was recognized that the presence of Wilson lines allows the Sivers functions to be non-vanishing [9, 11, 12, 13], also the remarkable result followed that the Sivers function in SIDIS differs by a sign from the one that enters in the Drell-Yan process. This process dependence is a unique prediction of QCD. It is entirely calculable, but has not been studied yet for other processes, such as $p\,p^{\uparrow} \to \pi\,X$ or the reaction $p\,p^{\uparrow} \to 2\,{\rm jets}\,X$ we have considered here. The color gauge invariant factorization is expected to make definitive statements here. Jet reactions may generally be easier to analyze theoretically than reactions with observed hadrons in the final state, since the latter involve also fragmentation functions that inevitably complicate the issue of gauge invariance further [9, 44]. A novel aspect in all this will also be the color gauge invariant definition of the gluon Sivers function, which has so far not been obtained (the same applies to any transverse momentum dependent gluon distribution and fragmentation functions). It will result in the proper gauge invariant version of Eq. (2). For our present study, the process dependence implies an uncertainty. We have refrained from making any ad hoc choices and simply used as a starting point the valence quark Sivers functions obtained from $p\,p^{\uparrow} \to \pi\,X$ [10].

It is evident that it will be very important to deal with these issues. A proof of a factorization theorem for the process $p\,p \to 2\,{\rm jets}\,X$ at small pair transverse momentum would be highly desirable. That said, even if factorization will be shown not to occur, the observable we have proposed is obviously still a quantity of interest. It will presumably then give us insight into novel aspects of QCD dynamics.

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